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Interpolation Polynomial Approximation for the Optimal Teaching of Employing Educational Technologies in Traditional Education of Mathematics

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ABSTRACT

Objective: In this paper, after having examined the concept of learning, we analyzed mathematics education using educational applications.

Methods: First, the stages of learning introduced. Next, learning stages are considered for the two modes of traditional method of mathematics education and using technology in the traditional education. These two methods were employed for different educational classes in some high schools of Karaj city. The data acquired were analyzed using data envelopment analysis.

Results: The inputs are traditional mathematics education and education through employing technology. The output is the four learning stages, which is also the second phase input. Learning is the output of the second phase.

Conclusions: we achieved an optimal ratio of using both methods in an educational class. Finally, we achieved an optimal ratio of employing both methods in a course and we find the internal polynomial.

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Introduction

Nowadays, many families are well aware of the significance of employing educational technologies in learning. The outbreak of epidemic diseases such as flu family, including corona virus, which has recently affected everyone in the world, highlights the necessity of using educational technologies in teaching more than ever. In the current situation, due to the emergence of corona virus, educational conditions have also changed in Iran (Malkhalifeh, 2020). Therefore, the focus has shifted from in-person toward virtual classes. This novel approach in education succeeds, providing the use of new technologies and teachers' abilities. The origin of virtual education goes back to ancient times and some scholars believe that it dates back to the time of Plato and his student, Dionysius. However, educational technologies are needed for virtual education (Watson, 2002).

The history of using technology in education goes back to the early nineteenth century (Watson, 2002). It was in this era that the importance of using technology in education became evident for educational experts. Employing technology was limited to visual aids, so that the first visual aids were used in America's schools in the form of films and slides (Watson,2002). In 1984, New Jersey Institute of Technology held the first courses of BA online. The innovator of the first virtual course was University of Phoenix, in 1989. University of California was founded as the largest technology-based education centre with 100 faculties and universities offering more than 1500 lessons based on printed materials, audio-visual and satellite programs, internet, and CDs(Loukas,2004).

In Iran, virtual education was established before the Islamic Revolution in Islamic Azad University and Payame Noor University. At the end of the 70's, the University of Tehran began using the virtual education software of LMS, which stands for Learning Management System. In 1380, the virtual education website of this university was set up providing 9 lessons for scholarship students. Employing electronic educational software has always been of importance in Iran's Education system, which is the biggest educational section of the country (Vitman, 2002). The outbreak of corona virus provided a turning point for employing educational software in Iran's learning environments. In Farvardin 1399 SH, the Student Education Network of Shad was launched by Iran's Education system. Through using this software, a new approach has been provided for teachers and students. Moreover, 14 million students of Iran and their families are involved using it(Bloom,2007).

Throughout the history, a variety of technologies have been used in mathematics education. Many researches have been conducted regarding the role of technology in mathematics education, often discussing the general advantages of technology. In this article, technology was employed in traditional method of teaching in order to determine the ratio of using technology in traditional method of mathematics education. Without the use of technology, some courses would be difficult to teach (Aminifar, 2007). Given the benefits of combining technology with traditional teaching, education can be promoted and facilitated. Technology is regarded as a support in enhancing skills, understanding mathematics concepts, solving problems, and developing mathematical reasoning. Educational technology presents this opportunity for students to show immediate feedbacks and as a tool provides accurate and fast visual representations of mathematical shapes. Moreover, it can be considered as a tool for students to practice mathematical shapes and as an opportunity to demonstrate the effects of these shapes. Educational technology can be viewed as an aid for the calculation of problems that cannot be done mentally and their main purpose is not practicing computational skills (Bishop,2002). Although various aspects of the impact of technology have been repeatedly emphasized, teachers should make thoughtful and appropriate use of educational technologies in mathematics education. Applying technology in a mathematics class should not lead into confusion and being limited to technical features. Students should consider this tool as a facilitator for solving mathematic problems(Loukas,2004).

Therefore, a question regarding the extent of using technology and mathematical software would be posed by every mathematics teacher (Bishop,2007). In this paper, we aim to determine an optimal ratio of employing educational technology in mathematics education using data envelopment analysis.

As an example, in traditional method of education, the following steps would be taken for drawing the graph of the function $f(x)=(2x-1)/(3x-1)$:

1. Determining the function's domain
2. Finding the critical points
3. Determining asymptote

4. Determining the direction of function changes
5. Determining some auxiliary points
6. Drawing the graph.

Following the instructions mentioned earlier, drawing a graph for a function such as $f(x) = e^x sin(x^2 + 1)$ would be a difficult task. In this case, having been taught the basic concepts, students would be led to employ an educational technology. Answering the question as to what extent and how to use software in order to prevent students becoming one-dimensional and using it just for mathematics problems is of great importance.

1. Educational technology

Educational experts use educational technology as a tool to tackle issues associated with education. It is evident that no curriculum has enough potential to fully teach students all subject areas and concepts related to a course of study. Therefore, employing educational technologies can assist in achieving the above-mentioned goal.

The definition of educational technology

Not only does educational technology refer to the use of audio-visual equipment, the development of television, radio, and educational machines, computer, and other educational tools, but also it depends on curriculum design and assessment, educational experiences, their implementation and reform. To put it more simply, it is a principal and logical way accompanied by a form of systematic thinking with a view to solving the problems of education and curriculum.

Employing educational technologies is a systematic method that implements and evaluates the entire teaching and learning process based on specified objectives and the curriculum of the education system. The aim of using educational technology in mathematics education is to achieve more effective education in order to increase the quality and quantity of learning.

2. Learning and its stages

Learning involves a function through which new or existing knowledge, behavior, abilities, and choices are perceived, reinforced, or altered. Learning does not occur immediately by itself, but it is promoted on the basis of prior knowledge. In the end, learning results in changes that are usually permanent.

Learning in humans can be part of the process of education, personal development, and exercises that may occur purposefully or through motivation. Learning results from studying and reflecting carefully on that which is being studied.

From the viewpoint of Ernest R.Hilgard, Marquis, et al. and some psychologists learning is defined as a relatively permanent change in one's emotions, thinking, and behavior that arises from experience. Behaviorists such as John Watson and Skinner, who considered humans' nature resilient, believed that learning plays the central role in growth.

Stages of Learning

Learning consists of the following four stages: 1. Unconscious incompetence 2. Conscious incompetence 3. Conscious competence 4. Unconscious incompetence

The first stage: Unconscious incompetence

In this stage, individuals do not have a skill and they are unaware that a skill gap exists. This is the worst stage for learners, since they assume they act correctly, without being aware of their lack of knowledge about many issues. Indeed, the fact that they do not know what they do not know would not propel them to learning.

For example, students in the stage of unconscious incompetence would act as follows regarding calculating the second derivative of the function $f(x) = x^2 sin(x)$) at the point $x = \pi$

$$f(x) = x^2 sin(x) f'(x) = 2x cos(x) f''(x) = 2\big(-sin(x)\big) f''(\pi) = -2 sin(0) = 0$$

This answer is wrong. This occurs for the reason that the students are unaware of their lack of knowledge about the derivatives of multiplied functions.

The second stage: conscious incompetence

In this stage, individuals do not have a skill; however, they are aware of the skill gap. In fact, they know that they do not know. Moreover, they notice that they should take action and improve their skills. It is in this stage that individuals understand the necessity of acquiring a skill.

For instance, students in the stage of conscious incompetence would act as follows in the case of calculating the twentieth derivatives of the function $f(x) = \frac{x^2}{x-1}$ at the point x=1/2:

$$f(x) = \frac{x\hat{}2}{x-1} = \frac{1}{x}$$

$$f'(x) = \frac{2x(x-1) - x^2}{(x-1)^2}$$

At the end, students would feel that it is a long solution and as a result would abandon continuing efforts for solving that. That is to say, they conclude that the solution is wrong. In other words, they know that they do not know.

The third stage: conscious competence

In this stage, individuals acquire a skill consciously. Indeed, they know that they know. This is the most difficult stage in which individuals consciously attempt to learn a skill and make it a habit. Moreover, due to its difficulties, they often cease to acquire that.

As an instance, in this stage, for calculating the second derivative of the function $f(x) = \sqrt{\frac{1}{x^2+1}}$ at the point x=0, students would firstly rewrite the function as $f(x) = (x^2+1)^{-1/2}$. Then, they would solve the rest of the problem.

The fourth stage: unconscious competence

In this stage, individuals acquire the ability to perform a skill and owing to constant practicing, they would do it unconsciously and out of habit. Therefore, it is in this stage that individuals do a skill unconsciously. Performing a skill in this stage requires expending much less energy and it would also be enjoyable.

For instance, students in the stage of unconscious competence would never do differentiation for calculating the second derivative of the function $f(x) = \frac{x-1)^3}{1+sin(x)}$ at the point x=1, they would provide the answer $f''(1) = 0$ without doing differentiation, instead. This occurs due to noticing $(x-1)^3$ and they know that the n derivative of x^{n+1} includes x. Thus, they would conclude that the second derivative of $(x-1)^3$ includes $x-1$ which the result would be zero for $x = 1$. It implies that students solve such problems unconsciously.

4.Mathematics education through Traditional method and through using technology

Different tools could be used in mathematics education. These tools can be categorized into two groups:

1. Using traditional tools such as chalks, blackboards, markers, whiteboards, posters, etc.
2. Using technologies such as recorders, projectors, slides, computers, advanced software, etc.

Using educational technology contributes to presentation and evaluation of course contents. Compared with traditional methods, using software in education provides teachers with the opportunity to teach a great deal of course contents more interestingly with spending less time. By applying educational technology, notably internet and educational networks and software, parents can continuously monitor all their children's activities and take actions to address their weaknesses. Technology plays a remarkably significant role for teachers in that not only does it facilitate common teaching tasks, but also enables them to create new learning activities and to design a learning environment in which even those students suffering from different incapability be able to learn like others.

Learning through educational technology consists of three basic parts:

1. The effect of educational technology on students' learning speed
2. The effect of educational technology on students' self-concept and their attitudes about learning
3. The effect of educational technology on student-teacher interaction

Comparisons that have been made between traditional mathematics education and technology-based education demonstrate that the application of technology has positive effects on learning. Using technology in science education has yielded similar results, too. This is evident in the case of a comparison made in a biology class of high school students who were divided into two groups. There was not a significant difference between the performance of participants who received simulated frog dissection on Disks and the other group that had real experience. This shows that at least learning some information through simulation can be as effective as learning through concrete and direct experiences. More importantly, it can be probably said that the group who received simulated dissection on Disc had more significant performance than the others who experienced dissection concretely(Uzel , 2006).

Among the most useful research findings on advanced educational tools, it seems that designing special software has a significant impact on the quality and quantity of students' learning. According to Bialo and Sivin-Kachala (1990) if students have some control over the quantity, review and sequence of education, they can progress more than the situation that all educational decisions are made by software. On the other hand, fully-structured educational guide is more suitable for low-achieving students. For instance, students of the fifth and sixth grade who had control over the time spent for each page in interactional education got higher scores compared with those who used the same software without having control over its steps(Dalton, 1990). Similarly, the improvement of students who used basic geometry software with the possibility to control over examples, exercises, and reviews were more than the students who used another version of the software without having such controls(Papanastasiou,2000). In addition, exercise software provide information about correct answers have greater effect on learning than those that require students' response for correction (Uzel , 2006).

A study on a group of university students reveals that students who gain knowledge required for a correct answer through studying social texts are much more successful than those who gain the correct answer after their own feedback (Papanastasiou,2000).

The results of the meta-analysis of Ryan (1991) on the effect of computer-based education on elementary students' improvement indicate that three main factors play a key role in making learning environments efficient:

1. Student-teacher and student-student interaction
2. Aligning educational technology with teacher-approved contents.
3. Assisting students in directing their learning and doing class activities.

The necessity and application of mathematics educational technology

The existence of numerous problems in mathematics education highlights the necessity of applying educational technology in order to solve such problems. Setting the stage for producing different mathematics education software is a principle of mathematics education at the international level. Given that technologies can spark the creative youth's interest in innovation, invention, and exploration, if schools were equipped with enough educational software, learning mathematics could be more interesting and engaging rather than boring.

The most important reason for using educational technology in mathematics education is the role of the five senses and the contribution proportion of each one in learning. According to studies, the proportion of each sense in learning is as follows:

75% of learning occurs through the sense of sight

13% of learning occurs through the sense of hearing

6% of learning occurs through the sense of touch

3% of learning occurs through the sense of smell

3% of learning occurs through the sense of taste

As reported by statistics, special attention should be paid by teachers to the percentages of the different five senses. Although students can learn one-fourth of what they hear, many teachers pay attention to the use of hearing sense (13%) and consequently about 60% of students' time in elementary schools and 90% in secondary schools are spent on hearing. The important point is that the use of technology by itself cannot enable us to achieve our goal, which is learning. However, there should be used an appropriate optimal ratio of the combination of traditional method of education and technology-assisted education by teachers.

Material and Methods

In implementing the method, seven schools of the same level in Karaj City were selected. The sample of the study were twelfth grade students of the above-mentioned schools. In each school, three subjects of calculus 2, geometry 3, and discrete mathematics were evaluated during four academic semesters. Next, 20 students of the same academic level were selected from each class using a pre-test. The data were normalized utilizing Kolmogorov Smirnov test. In addition, class scores attained a similar level in terms of performance with the help of the analysis of variance test (ANOVA), and the results indicated that there is no significant difference in their pre-test average scores. The teachers were selected in a way to have similar teaching experiences, to use the book recommended by the Department of Education, to have the same academic degree and teaching method. Schools were taught with different ratios of teaching using both the traditional method and technology. For instance, in one class, 0% of the teaching method was traditional, and 100% was technology-based. In another class, 20% of the teaching method was traditional, and 80% was technology-based and it was in this order with the last one in which 100% of mathematics education was traditional, and 0% was technology-based. In order to evaluate in each stage of teaching, four sets of questions were posed in a way to offer solutions as the acceptable answers for conclusion.

The results of the tests were combined in a table. In the present paper, only the condition, in which 60% of teaching was technology-based, and 40% was traditional, is shown in Table 1. In this table, the first column indicates the type of classes for twelfth-grade students in seven schools. The second column of the table shows the ratio of mathematics education using the traditional method in four educational semesters. The third column indicates the ratio of mathematics education using technology. Columns 4, 5, 6 and 7 respectively show the correct answers of 20 students in each class for four sets of questions.

Using multiplier output-oriented CCR model in data envelopment analysis the performance was evaluated. CRR model, which is among the most renowned models in data envelopment analysis, is as follows:

$$\max \varphi$$

$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{ro} \qquad r = 1, \dots, s$$

$$\lambda_j \geq 0$$

The input in the first phase is classified into traditional teaching method and technology-based teaching in percentage terms. The output of the first phase, which comprises the students' scores during the teaching process, is deemed as the input into the second phase. The output of the second phase is displayed in the three columns related to the final scores for the three courses of Calculus, Geometry and Discrete Mathematics.

This model was then used to examine data performance assessment. For example, if mathematics education using the traditional method is 40%, then technology-based teaching will be 60%. Therefore, the sum of traditional teaching of mathematics and technology-based education can be considered as 1. Accordingly, using the input-oriented CCR model won't be useful. For this purpose, the output-oriented CCR model was used. The outputs of the present paper refer to the results and achievements of students from the test, which ultimately examines learning.

Some of the important points for selecting this model in order to evaluate the performance of students in the three subjects mentioned above are the followings: (1) all students are evaluated together. (2) It is possible to examine in which tests and to what extent each student did not have the desired activity or understanding of a certain topic. Therefore, we can find their weaknesses and strengthen them. (3) In this model, all students are evaluated as a whole, so that the best and the worst results among those who took the tests are found, not merely the percentage of the correct answers of only one student. Therefore, in this type of assessment, the grade average or level will not cause any problem. Typically, in calculating the grade average or setting the level, the questions may have been designed in such a way that none of the students could have reached the adequate grade average or level. However, the results obtained from this model are related to all the students in the same condition.

Results

The results obtained from the above model will provide values greater than 1. Therefore, by finding the reciprocal of them from the multiplied model of CCR, the output nature of the results table will be in the range of 0 and 1. Moreover, since the results are limited in a bounded range of 0 to 1, they will become much clearer. It is noteworthy that efficiency 1 is the best and efficiency zero is the worst.

Table 1 shows the mean and variance of test results for both traditional education and technology-based education for four sets of questions. Table 2 indicates the number of correct answers supplied by students when 60% of mathematics education involves the traditional method and 40% is technology-based. Moreover, evaluation of performance using data envelopment analysis (DEA) was included.

Table 1. The mean and variance of test results for both traditional and technology-based methods for four sets of questions.

Tra	Tec	Level1		Level2		Level3		Level4	
		AVE	VAR	AVE	VAR	AVE	VAR	AVE	VAR
0	100%	15.55	1.418421	9.15	3.397368	7.30	1.063158	15.70	0.326316
20%	80%	15.70	21.37895	13.60	2.042105	13.95	1.628947	15.25	0.723684
40%	60%	13.15	1.923684	13.75	1.460526	13.30	1.589474	14.80	1.010526
50%	50%	14.65	0.344737	13.65	1.186842	12.90	1.463158	14.05	0.892105
60%	40%	16.15	1.186842	14.65	1.818421	13.90	1.568421	14.70	1.273684
80%	20%	13.95	1.523684	15.30	1.063158	16.55	0.681579	18.30	0.326316
100%	0%	5.10	1.252632	15.35	1.397368	17.05	2.997368	18.40	0.463158

Table 2. The Frequency of students' correct answers when 60% of mathematics education is traditional and 40% is technology-based and the evaluation of performance using DEA.

	Tra	Tec	L1	L2	L3	L4	Finla(C2)	Final(G)	Final(D)		DMU1	DMU2	DMU							
DMU121	0.6	0.4	15	15	12	14	16	16	16		0.865	0.800	0.692	218	0.865		224	0.800		0.692
DMU122	0.6	0.4	15	14	13	16	15	15	15		0.865	0.750	0.649	218	0.865		210	0.750		0.649

DMU123	0.6	0.4	17	15	15	13	17	17	17		0.948	0.850	0.806	239	0.948		238	0.850		0.806
DMU124	0.6	0.4	16	11	14	16	17	17	17		0.857	0.850	0.729	216	0.857		238	0.850		0.729
DMU125	0.6	0.4	16	14	13	15	20	20	20		0.893	1.000	0.893	225	0.893		280	1.000		0.893
DMU126	0.6	0.4	18	16	15	14	20	20	20		1.000	1.000	1.000	252	1.000		280	1.000		1.000
DMU127	0.6	0.4	16	14	15	17	18	18	18		0.917	0.900	0.825	231	0.917		252	0.900		0.825
DMU128	0.6	0.4	16	14	13	15	19	19	19		0.893	0.950	0.848	225	0.893		266	0.950		0.848
DMU129	0.6	0.4	17	16	12	14	20	20	20		0.944	1.000	0.944	238	0.944		280	1.000		0.944
DMU130	0.6	0.4	15	12	14	16	20	20	20		0.841	1.000	0.841	212	0.841		280	1.000		0.841
DMU131	0.6	0.4	17	15	14	13	18	18	18		0.940	0.947	0.891	237	0.940		252	0.947		0.891
DMU132	0.6	0.4	18	16	15	14	19	19	19		1.000	1.000	1.000	252	1.000		266	1.000		1.000
DMU133	0.6	0.4	15	15	16	14	19	19	19		0.922	1.000	0.922	226	0.922		266	1.000		0.922
DMU134	0.6	0.4	14	16	12	16	17	17	17		0.882	0.895	0.789	216	0.882		238	0.895		0.789
DMU135	0.6	0.4	16	16	14	15	18	18	18		0.959	0.947	0.909	235	0.959		252	0.947		0.909
DMU136	0.6	0.4	15	14	13	15	19	19	19		0.886	1.000	0.886	217	0.886		266	1.000		0.886
DMU137	0.6	0.4	17	16	13	15	17	17	17		0.984	0.944	0.929	241	0.984		238	0.944		0.929
DMU138	0.6	0.4	16	15	15	13	16	16	16		0.943	0.889	0.838	231	0.943		224	0.889		0.838
DMU139	0.6	0.4	17	15	14	15	17	17	17		0.976	0.944	0.921	239	0.976		238	0.944		0.921
DMU140	0.6	0.4	17	14	16	14	18	18	18		0.971	1.000	0.971	238	0.971		252	1.000		0.971

The frequency chart of the correct answers of all students is demonstrated in Figure 1. For example, in the first group of columns, 0% of teaching is traditional and 100% is technology-based. Likewise, in the last group of columns, 100% of teaching is traditional and 0% is technology-based. It is noteworthy that in order to analyze the data, we examined each class separately. In the present paper, we have presented merely the overall sum of the correct answers according to the ratio of teaching occurred in the relevant classes.

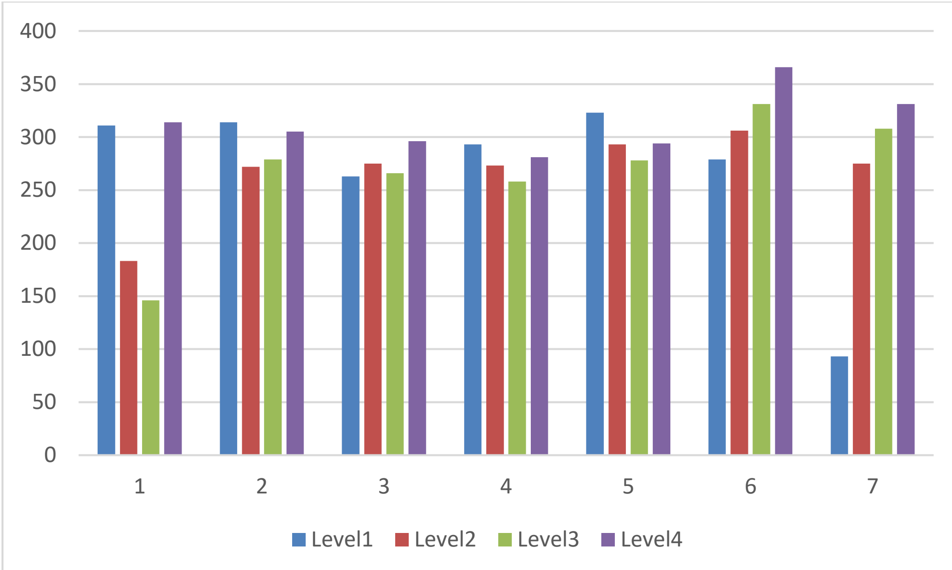

Fig. 1. The bar chart of students' correct answers classified by the ratio of education using either traditional or technology-based methods

According to the results obtained from the evaluation of students' performance, it was found that 12 classes out of the total of 84 achieved an efficiency of 1, i.e. the best performance. They demonstrated a minimum efficiency of 0.7 in these 12 classes, which indicates that most students are above the educational level of 70%, i.e. all students are in good condition in relation to the adopted teaching methods and in comparison, with all the other students. It can be said that by examining the condition of teaching and learning in addition to reviewing the obtained results, the academic level of students is almost uniform. Furthermore, 72 inefficient classes can be observed in order to achieve efficiency, or simply put the ideal level.

1. Interpolation polynomial approximation

Now, we want to provide an approximation for the training function using two methods of employing educational technologies in traditional education of mathematics by using the interpolation of bivariate functions, so that the input is the ratio of training expressed in the two methods and the output is efficiency. This approximation will help us to know what we can expect in learning if we consider the education to be, for example, 65% of traditional education and 35% of employing educational technologies. We use the Sheppard interpolation method to approximate the function.

Shepard's interpolation is a general method for interpolating multivariate functions.

Assume the following distinct nodal points,

$$p_i = (x_i, y_i), \quad 1 \le i \le n$$

have produced the values of $f(p_i)$, in fact, we must determine that the function $f: R^2 \rightarrow R$ converts the points p_i to $f(p_i)$.

Consider the real function $\varphi: R^2 \rightarrow R^2 2$ under the following condition:

$$\varphi(p, q) = 0 \quad \Leftrightarrow \quad p = q$$

where we used p and q to represent the elements in R^2. In this case, the polynomial of the Shepard interpolator is as follows:

$$F(p) = \sum_{i=1}^{n} f(p_i) u_i$$

which

$$u_i(p) = \prod_{j=1\ j \ne i}^{n} \frac{\varphi(p, p_j)}{\varphi(p_i, p_j)}, \qquad 1 \le i \le n$$

Now if we consider $\varphi(p, q) = \|p - q\|^2$, with considering $p = (x, y)$ and $p_i = (x_i, y_i)$ we will have:

$$\varphi(p, p_i) = \|p - p_i\|^2 = (x - x_i)^2 + (y - y_i)^2$$

Therefore, the interpolation polynomial is:

$$F(x, y) = \sum_{i=1}^{n} f(x_i, y_i) \prod_{j=1\ j \ne i}^{n} \frac{(x - x_i)^2 + (y - y_i)^2}{(x_i - x_i)^2 + (y_i - y_i)^2}$$

Now, we want to determine the efficiency of education and the interpolation polynomial for the two methods of educations using Shepard's interpolation.

The following table shows the frequency of correct answers to the tests given by some of the students in each class, by both methods. For four categories of questions along with performance evaluation using DEA. We want to determine the interpolation polynomial and the approximation for the other node points which is the ordered pair of the training ratio.

(x_i, y_i)	(0,1)	(0.2,0.8)	(0.4,0.6)	(0.5,0.5)	(0.6,0.4)	(0.8,0.2)	(1,0)
$f(x_i, y_i)$	0.387	0.495	0.590	0.659	0.872	0.614	0.461

Therefore, the polynomial in Shepard's interpolation is as follows:

$$f(x,y) \simeq P(x,y) = \sum_{i=0}^{6} f_i L_i(x,y)$$

where in

$$L_i(x,y) = \prod_{j=0\ j\neq i}^{6} \frac{(x-x_i)^2 + (y-y_i)^2}{(x_i-x_i)^2 + (y_i-y_i)^2}$$

To determine the approximation of f(0.55, 0.45) with the help of Shepard's interpolation polynomial, we used Maple software and obtained the following approximation:

$f(0.55,0.45) \simeq 0.8452$

Therefore, to teach 55% traditional education and 55% employing educational technologies, we will have an efficiency of 0.8452.

Discussion

Studies have shown that each of the five senses has a different role in learning. The most important reason for using educational tools is the role of the different senses in learning. Since applying audio-visual tools, educational software, and laboratory instruments are directly related to the five senses, notably the visual sense, they have high impact on learning.

Preparing the ground for the evolution of teaching and learning process and the use of technology with the aim of motivating students is considered as a curriculum design principle in education system.

Novel approach. Design and implement programs. Emphasizing on the continuity and students' seeking excellence in learning and nurturing educated people emphasizing on features such as having a questioning mind, curiosity, eagerness to learn, having multiple literacy, …. .

This study has shown the role of technology in learning and promoting students improvement. It is worth noting that the studies conducted in the field are limited in terms of number and research design. Moreover, these studies investigated a short period of educational technology history and several others have only concluded that this is a better type of education, since they evaluated technology-based approaches as complementary to education rather than alternative. However, recent studies confirm that the contribution of computer as an interactive educational technology is advantageous for students' learning. The use of technology in learning environments has demonstrated that how learning can be meaningful and continuous. Furthermore, it can lead to student-centered and collaborative learning and increase teacher-student interaction. Positive changes in learning resulting from technology led to evolution of this area. The results mentioned above demonstrate the intricacies of teaching and learning through advanced technologies that will occur in education throughout the next century. Also, it poses some challenges for educators who are faithful to promote learning in their own environments. Spending two decades on getting scientific experience of computer technology and one decade on doing research on it, provides this opportunity to make the following statements:

1. Although technology can improve teaching and learning, having it alone will not lead to a better educational efficiency. the fact that a certain school achieves the potential advantages of technology depends on what software is used, what is done with computer software and hardware by students, how computer-based learning is organized and supported by educators, and if enough technology is available.
2. To involve students in computer-based learning experience appropriately, and to lead educators create and support these experiences successfully, the educators' professional change is absolutely necessary, above all else.
3. To establish an appropriate education system and to decide upon using computer technology properly, choosing software, supporting and evolving professionally, founding hardware technology and all factors involved require quite coherent planning.
4. Computer and related technologies are frequently changing, hence, educational curriculums and their implicit components should also continually change and undergo evolution. Considering ascending changes of technology, new opportunities should be made frequently with the aim of the positive and innovative use of technology in education system. This paper establishes the foundation for designing curriculum and evolving and improving the present performance.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Ethics statement

The studies involving human participants were reviewed and approved by ethics committee of Islamic Azad University. The patients/participants provided their written informed consent to participate in this study.

Author contributions

K.N, M.R, M.M and H.R contributed to the study conception and design, material preparation, data collection and analysis. All authors contributed to the article and approved the submitted version.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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